ESTIMATING THE ROLE OF GOVERNMENT EXPENDITURE IN LONG-RUN CONSUMPTION

David Aristei and Luca Pieroni*

Abstract

In this paper we provide empirical evidence of the relationship between government purchases and private expenditure by adopting a microeconomic approach. Using UK quarterly data, a long-run demand system conditioned to the public sector is obtained by specifying a vector error correction model in which government consumption is assumed as an exogenous I(1) forcing variable. Our findings reject the hypothesis of separability of individual preferences between public and private expenditures, with simultaneous crowding out/in effects. Moreover, crowding out effects of government consumption on private spending are found to be larger for those goods and services that produce similar utility.

Keywords: Private and public expenditures, Crowding out, Conditional demand systems, Separability, Vector Error Correction Models.

JEL classification: D12, H31, H4

Acknowledgement: We are grateful to Carlo Andrea Bollino and Federico Perali for their useful suggestions and to Matteo Ricciarelli for his research assistance. All errors are our own.

* Department of Economics, Finance and Statistics, University of Perugia.

Corresponding author: Luca Pieroni, Department of Economics, Finance and Statistics, University of Perugia, via Pascoli 20, 06123 Perugia; Tel. +390755855280, Fax +390755855299, e-mail: lpieroni@unipg.it
1. Introduction

A central issue in macroeconomic analysis is testing the presence of a direct channel of influence of fiscal policy on private consumption decisions. The assumption that aggregate private consumption remains unaffected by a change in government expenditure has been proved to be questionable. Empirical investigations have shown that government expenditures exert a significant influence on private consumption behaviour (Kormendi, 1983; Aschauer, 1985; Kuehlwein, 1998). Moreover, when government consumption is decomposed into functional categories, the results concerning the impact of public expenditures are mixed, revealing the existence of both substitutability and complementarity effects (Graham, 1993; Karras, 1994; Kuehlwein, 1998; Fiorito and Kollintzas, 2004).

However, empirical macroeconomic literature has devoted little attention to the impact of government spending on the allocation of private consumption. As suggested by Aschauer (1993), a more profitable approach would also involve a decomposition of both private consumption and public spending in a way which might allow cleaner substitutability tests.

In this paper, a micro-based framework is used to derive a dynamic demand system conditioned to the public sector directly from a utility-maximization process, offering the opportunity to investigate the impact of different government expenditure changes on disaggregate private spending. Accordingly, we devote our attention to explaining how a conditional error correction model is twofold appropriate. Theoretically, the quantities of publicly provided goods and services are predetermined with respect to consumer choices. In order to specify the conditional (partial) model as a long run demand system, we conjecture that government expenditures are non-stationary so that public spending enters the cointegrated private demand system as an exogenous I(1) forcing variable (Johansen, 1992; Boswijk, 1994, 1995; Urbain, 1995; Harbo et al., 1998).

Moreover, the inclusion of government expenditures as structurally exogenous variables enables the long run substitutability effects with private expenditures to be recovered (Pieroni and Aristei, 2005). In accordance with the macroeconomic literature we can estimate a fixed substitution rate; however, coherently with time variant estimations of substitutability (Darby and Malley, 1996), the flexibility of the cost
function, which generates a first order approximation of the private demand system, enables us to consistently recover the estimated effects for each period of the sample.

In the empirical section, we use quarterly UK data for the period 1964Q1-2002Q2, adopting a functional classification of private expenditures to obtain a demand system which includes three macro-categories of goods: 1) Health, Education, Social Protection, Recreation and Culture (HER); 2) Other Services; 3) Food, Energy and other non-durables. Government expenditures are firstly represented by “Total public consumption” (G), which is then decomposed into “Individual public consumption” (GI), and “Collective public consumption” (GC).

The main objectives of the analysis is to test whether private decisions are affected by government allocation and to simultaneously verify the presence of crowding out/in effects in different categories of private spending. In particular, it is worth investigating how the HER category responds to changes in government allocations since the relationship between those private and public categories that produce similar utility has not been exhaustively tested. Finally, in order to obtain a consistent comparison with the macroeconomic literature, we build an aggregate indicator of crowding out starting from the substitution elasticities of government expenditures on the disaggregate demand system.

The rest of the paper is organized as follows. Section 2 outlines the conditional demand system theory, while in Section 3 the partial vector autoregressive model, connected with the specification of the conditional long-run Almost Ideal demand system, is discussed. In Section 4 we present the empirical results. Section 4.1 discusses the data and the time series properties. Section 4.2 reports the estimations of conditional long run demand system, while Section 4.3 assesses the relationships between private and public expenditures. Section 5 concludes the paper.

2. Theory

A formal way of testing the impact of the public provision of goods and services is represented by the specification of a flexible demand system, extended to the public sector, in which government expenditures enter consumer utility function in a non-
separable way. In empirical demand studies, the utility conferred on consumers by publicly provided goods and services is usually disregarded and separability of individual preferences between private and public consumption is implicitly assumed.

Unlike private expenditures, the public provision of goods and services, although directly conferring utility on the consumer, is not freely chosen but established a priori by the government; the level and composition of public spending, as well as the taxes needed to finance those expenditures, are therefore exogenous from the consumer viewpoint. Given this setting, the theory of consumer behaviour under quantity constraints (Pollak, 1969, 1971; Neary and Roberts, 1980; Deaton, 1981; Deaton and Muellbauer, 1981) can be applied to derive demand functions for private goods conditioned to publicly provided items and to analyse the influence of public consumption on private expenditures.

In order to derive the conditional demand functions, following Pollak (1969), we define a class of freely purchased goods whose \( n \times 1 \) vectors of quantities and prices are denoted as \( y \) and \( p \), respectively. We then define the class of rationed goods whose preferences are not parameterised and whose consumption is pre-determined in the quantity \( z \) at the fixed price \( r \). The cost function \( c^*(u, p, r, z) \) can be defined as the minimum cost needed to obtain the utility level \( u \), given the price vectors \( p \) and \( r \) when the quantity \( z \) of the rationed good must be purchased. Formally:

\[
c^*(u, p, r, z) = \min_y \left[ r' z + p' y | u(y, z) = u^*, z = z^* \right]
\]

\[
= r' z^* + \min_y \left[ p' q | u(y, z) = u^* \right]
\]

\[
= r' z^* + \gamma(u, p, z^*)
\]

where \( \gamma(u, p, z^*) \) is the conditional cost function. From the cost function [1], a conditional demand system can be derived. Firstly, it should be noted that the price of the rationed good \( r \) enters the cost function only through the fixed term \( r' z^* \); for this reason the conditional compensated (Hicksian) demand functions, obtained as the gradient of \( c^*(u, p, r, z) \) with respect to \( p \), do not depend on \( r \):

\[
\frac{\partial c^*(u, p, r, z)}{\partial p} = h(u, p, z) = y
\]
Inverting the cost function [1] and substituting it into the compensated demands [2], we obtain the conditional uncompensated (Marshallian) demand functions, which relate \( y \) to prices \( p \), to total expenditure \( e \) and to the quantity of the ration \( z \):

\[ y = g(e, p, z) \]  

where \( q \) represents the \( n \times 1 \) partitioned quantity vector of the freely purchased goods.

This framework can be adopted to analyse the relationships between private and public consumption. Denoting as \( Y \) the \( n \times 1 \) vector of privately purchased goods and as \( G \) the \( m \times 1 \) vector of quantities of publicly provided goods and services, the consumer utility maximization problem becomes:

\[ \max [u = u(Y, G)] \quad \text{s.t.} \quad YP = E \]  

where \( P \) represents the vector of prices of the freely chosen goods, \( G \) denotes publicly provided goods and services and \( E \) equals disposable income, since \( G \) is assumed to be entirely financed by tax revenue. Solving the constrained maximization problem, we derive the following uncompensated conditional demand function:

\[ Y = g(E, P, G) \]  

The public provision of goods and services exerts two effects on the demand for privately purchased goods: an income effect, whereby an increase in the provision financed by taxing reduces the amount of income available to purchase freely chosen items, and a substitution/complementarity effect, whereby the consumer rearranges his expenditures on freely chosen goods following a change in the quantity constraint.

Therefore, the standard demand function \( Y = g(E, P) \) is simply a special case of function [5], which is only correct when public consumption \( G \) is separable from private expenditures \( Y \). Our aim is therefore to verify the possibility of restricting the conditional demand function [5] and to test for separability between private and public consumption.

In order to derive the separability test, we have to first define a specific functional form in which public consumption enters the utility function in a non-separable way, otherwise, by construction, public consumption will not influence consumer behaviour and the effect of public purchases will be reduced to an income effect only, with private expenditures decreasing as government spending, and hence its financing through
taxation, grows (Tridimas, 2002). Moreover, the functional form of the demand functions should be as flexible as possible in order to analyse all the possible interactions between private and public expenditures (Levaggi, 1998). The Almost Ideal Demand System (Deaton and Muellbauer, 1980) seems to be the appropriate specification, since it provides a first order approximation of any demand system and allows a wide range of substitution effects. Following Tridimas (2002), the cost function of the conditional Almost Ideal model can be written as:

$$\log C(u, P, G) = \alpha_0 + \sum_{i=1}^{n} (\alpha_i + \sum_{j=1}^{m} \theta_{ij} G_j) \log P_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{k=1}^{m} \gamma_{ik} \log P_i \log P_k + u(Y, G) \phi_0 \prod_{i=1}^{n} p_{ij}^\phi \ [6]$$

Minimizing the cost function $C(u, P, G)$, given the market prices of the privately purchased goods, yields the demand equation in terms of budget shares:

$$w_i = \alpha_i + \sum_{k=1}^{n} \gamma_{ik} \ln P_k + \phi_j \left[ \log E - \log P \right] + \sum_{j=1}^{m} \theta_{ij} G_j \ , \ i, k = 1, 2, ..., n \ , \ j = 1, 2, ..., m \ [7]$$

where $\gamma_{ik} = (1/2)(\gamma_{ik}^* + \gamma_{ik}^*)$ , $P_k$ is the relative price of the $k$-th good, $X$ represents total per capita expenditure, $\log P = \alpha_0 + \sum_{i=1}^{n} (\alpha_i + \sum_{j=1}^{m} \theta_{ij} G_j) \log P_i + (1/2) \sum_{i=1}^{n} \sum_{j=1}^{m} \gamma_{ik} \log P_i \log P_k$ is a functional form usually approximated by the Stone index ($\sum_{i=1}^{n} w_i \log P_i$) and $G_j$ represents real per capita public expenditures. The theoretical constraints of adding-up, homogeneity and symmetry imply the following restrictions directly on the parameters of the model:

Adding up: $\sum_i \alpha_i = 1$; $\sum_i \gamma_{ik} = 0$; $\sum_i \phi_j = 0$; $\sum_i \theta_{ij} = 0$, $\forall k, j$ \ [8]

Homogeneity: $\sum_i \gamma_{ik} = 0$, $\forall i$ \ [9]

Symmetry: $\gamma_{ik} = \gamma_{ki}$, $\forall i, k$ \ [10]

Unlike homogeneity and symmetry, the adding-up constraint is endogenously satisfied. The validity of restrictions (2) makes the variance-covariance matrix singular; to solve this problem, an arbitrary equation must be omitted from the system, so that only the $n-1$ non-singular equations are estimated.

Given this setting, separability of consumer preferences between private and public goods can be verified by checking the statistical significance of the $\theta_{ij}$ parameters.
Moreover, the introduction of $G$ as an additional element of equation [7] further allows us to measure the direct effect of public consumption on each category of private spending as the percentage change in the demand for $Y_i$ following a unitary change in $G_j$ ($e_{ij} = \theta_{ij} / w_i$).

3. Econometric model

In this section we define the properties of a partial VAR model in order to estimate the relationship between private and government expenditures. As in the previous static demand system, government expenditure variables are modelled as conditioning goods. The motivation to use a conditional subsystem in demand analysis stems from the traditional way one usually partitions the set of variables under investigation, $x_t$, between the so-called endogenous variables, $y_t$, and those whose generating process will not be modelled, the exogenous variables $z_t$.

Formally, a VAR($p$) is considered for the $m \times 1$ vector of variables $x_t$:

$$A_0 x_t = a_0 + a_1 t + A_1 x_{t-1} + \ldots + A_p x_{t-p} + \xi_t, \quad t = 1, 2, \ldots, T$$

where $a_0$ is a constant term, $a_1$ is the coefficient of the deterministic trend, $A_i$ ($i = 1, 2, \ldots, p$) is a $m \times m$ matrix of unknown parameters and $A_0$ is a non-singular matrix. Empirical evidence suggests that both private consumption variables and public consumption expenditures are non-stationary (Sturm, 1998; Lewbel and Ng, 2000; Pesaran and Shin, 2002), so that cointegration in private expenditure categories is a necessary condition for estimating a long-run dynamic demand system. Thus, it is convenient to rewrite the VAR($p$) (11) as a vector error correction model (VECM):

$$\Delta x_t = b_0 + b_1 t + \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \ldots + \Gamma_{p-1} \Delta x_{t-p} + \epsilon_t$$

where $\Pi$ is the long-run impact matrix which describes the long-run relationships among the variables and $\Gamma_i$ are the short-run impact matrices, which account for the effects of short-run dynamics. The other elements of equation (12) represent the
constant term \((b_0 = A_0^{-1}a_0)\), the deterministic trend \((b_1t = A_0^{-1}a_1t)\) and the disturbances \(\varepsilon_t = A_0^{-1}\xi_t\) (with \(\varepsilon_t \sim iid(0, \Lambda)\)).

In order to rewrite (12) as a conditional error correction model, we partition the \(m \times 1\) vector \(x_t\) into the \(n \times 1\) vector \(y_t\) and the \(k \times 1\) vector \(z_t\), that is \(x_t = (y_t', z_t')'\), \(t = 1, 2, ..., T\). By partitioning the error term \(\varepsilon_t\) (and its covariance matrix) conformably to \(x_t\) as \(\varepsilon_t = (\varepsilon_t', \varepsilon_t')'\), it is possible to express \(\varepsilon_{yt}\) conditionally on \(\varepsilon_{yt}\) as:

\[
\varepsilon_{yt} = \Lambda_{zz}^{-1}z_t\varepsilon_{yt} + u_t \tag{13}
\]

where the innovations \(u_t\) are distributed as \(N(0, \Lambda_{uu})\), with \(\Lambda_{uu} = \Lambda_{yy} - \Lambda_{yz}\Lambda_{zz}^{-1}\Lambda_{zy}\), and are independent of \(\varepsilon_{yt}\). Substituting (13) into (12), together with a similar partitioning of the other vectors and matrices of parameters\(^1\), and assuming the process \(\{z_t\}_{t=1}^{\infty}\) as weakly exogenous with respect to the long-run impact matrix \(\Pi\), \(i.e.\) \(\Pi_x = 0\), we obtain a conditional long-run structural matrix \(\Pi_{yy, z} = \Pi_y\).

Rearranging the parameters, we obtain the conditional and marginal equations, respectively:

\[
\Delta y_t = \omega_0 + \omega_1t + \Psi\Delta z_t + \sum_{r=1}^{\rho-1} \Phi_{zr}\Delta x_{t-r} + \Pi_y x_{t-1} + u_t \tag{14}
\]

\[
\Delta z_t = b_{z0} + \sum_{r=1}^{\rho-1} \Gamma_{zr}\Delta x_{t-r} + \varepsilon_{zt} \tag{15}
\]

where \(\omega_0 = -\Pi_y a_0 + (\Pi_y - \Lambda_{yz}\Lambda_{zz}^{-1}\Gamma_y + \Pi_y) a_1\), \(\omega_1 = -\Pi_y a_1\) and \(\Phi_{zr} = \Gamma_{zr} - \Lambda_{yz}\Lambda_{zz}^{-1}\Gamma_{yr}\).

The restriction \(\Pi_z = 0\) clearly excludes cointegrating relationships in the marginal model (15). Moreover, this restriction makes the information available from model (15) redundant for efficient estimation and inference on \(\Pi_y\) as well as on \(\omega_0\), \(\omega_1\), \(\Psi\) and \(\Phi_{zr}\).

Thus, in line with Granger and Lin (1995), we define \(\{z_t\}_{t=1}^{\infty}\) as long run forcing for \(y_t\).

\(^1\) The vectors and matrices of parameters in equation (12) are partitioned as: \(b_0 = (b_{0y}, b_{0z})'\), \(b_1 = (b_{1y}, b_{1z})'\), \(\Pi = (\Pi_y, \Pi_z)'\), \(\Gamma = (\Gamma_y, \Gamma_z)'\)
By Granger’s theorem of representation constrained to the conditional model (14), the $\Pi_y$ matrix is decomposed as $\Pi_y = \alpha_y \beta'$ by the $n \times r$ loadings matrix $\alpha_y$ and the $m \times r$ matrix of cointegrating vectors $\beta'$.

The cointegration rank hypothesis is tested in the context of (14) as:

$$H_r : \text{Rank}[\Pi_y] = r, \quad r = 0, \ldots, n.$$  \[16\]

As it is known, the results of the cointegration rank test strictly depend on the specification of deterministic components of the model; the choice of the appropriate specification is often motivated by theoretical suggestions.

The assumption that $Z_t$ is weakly exogenous for $\beta'$ enables us to make inference on the cointegrating vector from the conditional model (14) since it is assumed that the variables $Z_t$ do not react to disequilibria. Note that when formulating a VAR model the selection of the variables is usually influenced by economic theory. In the context of a long run demand system, government expenditures are modelled as a structurally exogenous random variable, since the public provision of goods and services is predetermined with respect to consumer choice, which is assumed to have a I(1) data generating process. Thus, given this specification, the parameters of interest for the conditional demand system are those included in the cointegrating vectors. Disregarding deterministic terms, the $r$ cointegrating vectors of the AI model can be written:

$$cv(r) = \beta' \mathbf{x}_{t-1} = \beta'(w_{t-1}, \ldots, w_{n-1}, \ln p_{t-1}, \ldots, \ln p_{m-1}, \ln(E_{t-1}/P_{t-1}); \mathbf{G}_t)$$  \[17\]

Moreover, a variety of theoretical hypotheses, including substitutability effects between private and public consumption and homogeneity and symmetry properties, can be tested by imposing restrictions directly on $\beta'$.

Concerning the identification of the long run demand system, the adding-up constraint is a crucial theoretical assumption. Firstly, a sufficient condition to recover a structural demand system from the data generating process is that the number of cointegrating relationships should be equal to the number of non-singular demand equations ($r = n - 1$) specified in terms of budget shares ($w_j$). The rank condition excludes all the cases in which $r < n - 1$. With respect to the long run estimation of private demand systems (Pesaran and Shin, 2002), the rank test of cointegration of the
\( \Pi \) matrix is performed by introducing government expenditures as conditioning \( I(1) \) variables. The critical values used for the rank cointegration test with exogenous \( I(1) \) are taken from Harbo et al. (1998).

The estimation of the VEC model (14), subject to reduced rank restrictions on the \( \Pi \) matrix, does not lead to the exact identification of the cointegrating relations. Given the decomposition \( \Pi = \alpha, \beta' \), the identification of the parameters in \( \beta' \) requires the imposition of at least \textit{r a priori} restrictions on each cointegrating vector\(^2\). A necessary and sufficient condition for the identification of the long-run parameters is that \( \text{rank}\{R(I_1 \otimes \beta)\} = r^2 \); this condition holds provided that the number of the identifying restrictions, \( k \), is at least equal to \( r^2 \) (order condition).

Thus, the exact identification of the cointegrating relationships of the long-run AI demand system in \( n \) equations requires \( r^2 = (n - 1)^2 \) restrictions on the parameters of the cointegrating vectors, which are given by:

\[
H_{ki} = \begin{bmatrix}
\beta_{11} = -1 & \beta_{12} = 0 & \cdots & \beta_{1(n-1)} = 0 \\
\beta_{21} = 0 & \beta_{22} = -1 & \cdots & \beta_{2(n-1)} = 0 \\
\vdots & \vdots & \ddots & \vdots \\
\beta_{n-11} = 0 & \beta_{n-12} = 0 & \cdots & \beta_{n-1(n-1)} = -1
\end{bmatrix}
\]

[18]

Moreover, \( k - r^2 \) over-identifying restrictions can be imposed and tested; in the present application these restrictions may be taken directly from demand theory. The empirical validity of the theoretical constraints of homogeneity and symmetry can therefore be verified using the log-likelihood ratio statistic \( \chi^2 = 2\{l(H_{ei}) - l(H_{ii})\} \), which is asymptotically distributed as a \( \chi^2 \), with degrees of freedom being equal to the number of over-identifying restrictions imposed.

Finally, government expenditures enter demand functions as additional forcing variables, whose legitimate exclusion can be justified only if private commodity demands are weakly separable from government expenditures. Separability is proved when the parameter(s) of \( G_i \) in the \( \beta' \) vector are not statistically significant in

\(^2\) Pesaran (1997) proposes a particular approach for the identification of the long-run coefficients, which emphasises the importance of economic theory, suggesting that identification restrictions should be derived from an underlying economic model.
modifying private consumption decisions. Consequently it is possible to restrict the conditional model to a private demand system. On the contrary, if separability of individual preferences is rejected, the conditional specification of a long run demand system is appropriate.

To sum up, the demand model used in this work is characterized by the following economic properties: a) the conditional model leads only private budget shares to adjust in response to long-run deviations from equilibrium; b) demand theory imposes the theoretical restrictions of homogeneity and symmetry directly on the long-run parameters of the cointegrating vector $\beta'$, c) the government expenditure $G_t$ represents an exogenous I(1) forcing variable and, from a theoretical point of view, it allows us to account for substitutability effects by testing the separability hypothesis from a nested private demand system. In order to distinguish different impacts on the relationships between private expenditure and government consumption, in the next section we will implement different conditional models and the separability test is specified for aggregate government expenditure as well as for individual government and collective government expenditures.

4. Empirical evidences

4.1. Data and Time Series Properties

The long-run conditional Almost Ideal model presented in the previous sections is estimated for a three-commodity demand system using UK quarterly data. The choice of a high aggregated representation of data is connected to the data-intensive nature of the VECM approach, that prevents the correct analysis of demand systems with a large number of consumption categories. The three private consumption categories that make up the demand system are: 1) Health, Education, Social Protection, Recreation and Culture; 2) Services (including rents and rates); 3) Food, Energy and other non-durables. The main feature of this functional classification consists in the aggregation of private expenditures on health, education, recreation and culture and social protection in a single category. The definition of this particular category is aimed at the aggregation
of all household expenditures on those goods and services which can also be offered by
the public sector.

The private demand system is conditioned by including public expenditures as
rationed quantities in order to test for separability of individual preferences between
private and public consumption items. The Almost Ideal model is firstly conditioned to
total public consumption (G) so as to evaluate the overall effect of the public provision
of goods and services on private consumption behaviours. Aggregate public
consumption is then divided into the categories “Individual public consumption” (GI),
which includes public expenditure on health, education, social protection, recreation and
culture, and “Collective public consumption” (GC), which includes government
expenditures on defence, public order and safety, general public services and justice.
This representation of public expenditures allows us to separately analyse the effects of
those goods and services which are non-rival and non-excludible in consumption from
those which were more akin to private goods (Fiorito and Kollintzas, 2004).

The data used in the empirical application are taken from the UK quarterly National
Accounts, published by the Office for National Statistics (ONS)\(^3\). The series of private
consumption expenditures are seasonally adjusted and expressed both at current and
constant 2002 prices; price indexes for each commodity group are obtained as implicit
deflators of the three expenditure categories. Private spending is net of durable
consumption expenditures, which are included in some non-food spending
components\(^4\). Total expenditure is transformed into per capita terms, by dividing the
original series by resident population\(^5\).

Public consumption expenditures are expressed at constant 2002 prices and
transformed into per capita terms. For the aims of our analysis, an issue with public
expenditure data is that quarterly observations are available for the series of total public

---

\(^3\) More precisely, private expenditure data are taken from “*Household final consumption expenditure: classified by purpose*” (Blue Book, Tables 6.4 and 6.5), while those concerning government spending are obtained from “*Gross domestic product: expenditure approach*” (Blue Book, Table 11.2).

\(^4\) The presence of durable components within private consumption may represent a problem for a correct specification of the demand system, because of the temporal gap between the moment in which those expenditures are made and when they transfer their utility. Supporting arguments for the exclusion of durable expenditures can be found in several studies (Aschauer, 1985; Marrinan, 1998; Tridimas, 2002).

\(^5\) Quarterly data on population are obtained by interpolation of annual population figures published by ONS.
consumption only, while disaggregated information on the composition of public spending is available on annual basis. Quarterly data on real individual and collective public consumption are obtained by applying the Chow-Lin temporal disaggregation technique (Chow and Lin, 1971, Fernandez, 1981) to the relative annual series\(^6\). In particular, the quarterly series of aggregate public consumption is used as the indicator variable, under the hypothesis that low-frequency series movements within the period follow the pattern of total public spending. The estimated quarterly series are then adjusted for seasonality by means of the Census X12-ARIMA method.

Since the cointegrated VAR analysis pre-assumes the variable of the demand system to be non-stationary, Augmented Dickey-Fuller (Dickey and Fuller, 1979) and Phillips-Perron (Phillips and Perron, 1988) tests are computed to examine whether the time series of the three budget shares \((w_1, w_2, w_3)\), of the relative prices \((\log p_1, \log p_2, \log p_3)\), of real total expenditure \((\log E - \log P_t)\) and of real government expenditures \((G, GI\) and \(GC)\) contain a unit root in their data generating process. Table 1 summarizes the results of the ADF and PP tests. Since the results are sensitive to the inclusion of deterministic components in the test regression, we compute the unit root tests including both a constant and a linear trend and only a constant in the underlying regressions; the inclusion of irrelevant deterministic components may in fact reduce the power of the test to reject the null of a unit root. Moreover, the number of lagged difference terms in the ADF test regression has been selected by using Schwarz information criterion so as to remove serial correlation in the residuals. The results of both the specifications of the ADF and PP tests clearly indicate that it is not possible to reject the null hypothesis of unit root at the 5% and 1% significance levels for any of the variables of the demand system.

*Table 1 about here*

\(^6\) The methodology used is based on the work of Chow and Lin (1971), Fernandez (1981) and Litterman (1983). Details on the annual to quarterly disaggregation of individual and collective public consumption are available from the authors.
4.2. Estimating the Long-run Demand System

In this section the conditional demand system (14) is estimated using alternative definitions of public consumption. In particular, assuming non-separability between private and public goods, two conditional specifications are considered, obtained by alternatively including total public consumption (G) and individual (GI) and collective (GC) government expenditures as exogenous I(1) forcing variables.

In the estimation of the conditional model, the adding-up restrictions are not testable and are *a priori* imposed, requiring a budget share equation to be omitted from the system. The results are invariant to the choice of the $n-1$ equation included in the model; in this analysis the budget share equation of Food, Energy and other non-durables ($w_3$) is omitted and its long-run parameters are then determined from the adding-up constraints.

In order to verify the presence of cointegrating relationships between the variables of the demand system, it is necessary to solve some preliminary issues connected with the specification of the VEC model (14). Firstly, we have to define the order of the autoregressive model. The relatively short length of the sample size does not allow the use of a large autoregressive dimension; moreover, as Boswijk and Franses (1992) and Reimers (1992) show, the selection of the order $p$ is crucial in the VECM specification since it may affect the cointegration rank test. In particular, an excessive number of lags lowers the power of the test, while an underspecification of the VAR order could potentially lead to the much more relevant problem of spurious cointegration. The lag order is selected by using the Akaike Information Criterion (AIC), the Schwarz Bayesian Criterion (SBC) and a small-sample corrected likelihood ratio test\(^7\). The results are reported in Table 2; as it can be noted, the two information criteria considered and the adjusted LR test univocally indicate that the optimal lag order, for both the conditional specifications, is equal to two ($p = 2$).

\(^7\) In selecting the lag order, we do not consider orders higher than 6, because of data limitations.
Since the cointegrating rank hypothesis depends on deterministic variables, the reduced rank procedure of Johansen is revised to estimate the conditional system with these variables included (Pesaran et al., 2001). Consumer theory predicts that budget shares converge towards a steady state value proxied by the constant $\gamma_0$; thus, we assume a VAR(2) model with restricted intercepts and no trend to ensure that steady state values for the budget shares exist both under the null and the alternative hypotheses (Pesaran and Shin, 2002). Formally, the structural VECM estimated becomes:

$$\Delta y_t = (-\Pi y_0) + \Psi \Delta z_t + \Phi_1 \Delta y_{t-1} + \Pi_1 x_{t-1} + u_t$$  \hspace{1cm} [18]$$

where $y_0$ is the intercept term of the cointegrating relationships.

Only when the model specification is defined, it is possible to determine the rank of the long-run multiplier matrix, $\Pi y_0$; the asymptotic distribution of the LR test statistic for cointegration does not have standard distribution and strictly depends on the assumptions made with respect to both the lag length and the deterministic components of the model. The test for the presence of cointegrating relationships among the private demand variables is developed by using the maximal-eigenvalue and trace tests (Johansen, 1995), in which the critical values are modified to take into account the exogenous I(1) forcing variables (Harbo et al., 1998). The necessary condition to identify an error correction demand system requires two cointegrating relationships among the six endogenous variables of the conditional demand system, corresponding to the two non-singular budget share equations. The results of the cointegration tests are presented in Table 3. At the 5% significance level, the maximal-eigenvalue and the trace statistics unambiguously indicate the presence of two cointegrating relationships, providing support for the definition of the conditional cointegrated demand system.

Table 3 about here

Thus, the necessary condition for the exact identification of the parameters of the two cointegrating vectors requires four restrictions to be imposed; these exact identifying
restrictions, implicit in the specification of the share equations of the AID model, take the following diagonal structure:

$$H_{EI} = \begin{cases} 
\beta_{11} = -1 & \beta_{12} = 0 \\
\beta_{21} = 0 & \beta_{22} = -1 
\end{cases}$$  \[19\]

The exactly identified matrix of long-run parameters will then have a total of five and six unrestricted parameters to be estimated, respectively, for the demand systems conditioned to total public consumption and to individual and collective public expenditures. These unrestricted parameters corresponds to the three prices, to total per-capita expenditure and to the public consumption variables (G or GI+GC, alternatively), plus the constant terms. The exactly identified estimates of the two cointegrating vectors are presented in Table 4\(^8\); the maximized values of the log-likelihood functions of the exact identified VEC models, conditional on G and GI+GC, are equal to 3709.8 and 3715.2, respectively.

Table 4 about here

In order to verify the coherence of the statistical model with demand theory, the homogeneity and symmetry constraints are imposed as over-identifying restrictions and empirically tested. In particular, the homogeneity constraint implies the following two restrictions on the long-run parameters:

$$H_{hom} = \begin{cases} 
\beta_{13} + \beta_{14} + \beta_{15} = 0 \\
\beta_{23} + \beta_{24} + \beta_{25} = 0 
\end{cases}$$  \[20\]

while the symmetry constraint requires the following cross-equation restriction on the parameters of the two cointegrating vectors:

$$H_{sym} = \{\beta_{14} = \beta_{23}\}$$  \[21\]

\(^8\) Using the adding-up constraints, the third cointegrating share equations are given by:

$$\hat{\omega}_3 = 0.4522 - 0.2010 \log p_1 + 0.1502 \log p_2 + 0.0181 \log p_3 - 0.3052 \log (E_t/P_t) - 0.0335 G_t$$

and

$$\hat{\omega}_3 = 0.4896 - 0.2150 \log p_1 + 0.1502 \log p_2 + 0.0323 \log p_3 - 0.2777 \log (E_t/P_t) - 0.0372 GI_t - 0.0285 GC_t$$

for the demand systems conditioned to G and GI+GC, respectively.
The maximized values of the log-likelihood functions for the two demand systems conditioned to G and GI+GC, obtained by imposing both the theoretical restrictions, are equal to 3703.3 and to 3707.9, respectively.

The empirical validity of the theoretical constraints of homogeneity and symmetry is tested using a likelihood ratio (LR) test, based on the log-likelihood values of the restricted and unrestricted specifications. The test statistic, which is asymptotically \( \chi^2 \) under the null, is defined as \( LR = 2(\log L_{UR} - \log L_{R}) \). In finite samples standard asymptotic results can be very misleading, since they are biased towards over-rejection when the number of equations and parameters of the models is large with respect to the sample size (Dufour and Khalaf, 2002). In particular, testing homogeneity and symmetry in demand systems has proved to be very sensitive to this problem, especially when models are heavily parameterized (Laitinen, 1978; Pudney, 1981; Theil and Fiebig, 1985). In this analysis, it is therefore worth implementing a small-sample adjustment to correct the over-rejection tendency of the LR tests. Following Pudney (1981), we define the adjusted LR statistic as follows:

\[
LR' = LR + nT \log \left[ \frac{nT - p_1}{nT - p_0} \right]
\]

where \( n \) is the number of equations and \( p_0 \) and \( p_1 \) are the numbers of parameters of the restricted and unrestricted specifications, respectively. An analogous correction is also carried out for the critical values, which take the form \( K = nT \log \left[ 1 + \frac{dF_{nT-p_1}^{d}}{(nT - p_1)} \right] \), where \( d = p_1 - p_0 \) and \( F_{nT-p_1}^{d} \) is the critical value for the \( F \) distribution.

The results of the joint test of homogeneity and symmetry, with and without small sample correction, are presented in Table 5. For comparability purposes, theoretical restrictions have also been tested for the separable demand system, so as to obtain a comprehensive check of the consistency of demand theory with the data.

Table 5 about here

As it is common in demand studies, the joint asymptotic test for the theoretical restrictions leads to a general rejection of the null hypothesis in all the three
specifications considered, with LR test statistics well above the corresponding 1% critical value. As it can be noted, after applying the small-sample correction, the test results slightly change, with an evident improvement in the statistical significance of the homogeneity and symmetry constraints for the model conditioned to total public consumption. In particular, the small-sample adjusted LR tests indicate that the theoretical restrictions can be rejected at the 5% significance level for all the specifications considered, even if we are unable to reject homogeneity and symmetry restrictions at the 1% level of significance in the conditional specifications.

These results underline that the consistency of the theoretical restrictions with the data appears to be questionable; hence, we proceed by a priori imposing homogeneity and symmetry. The estimates of the two cointegrating vectors, subject to the joint imposition of the restrictions (20) and (21), are presented in Table 6.

Table 6 about here

Analyzing these results, it is possible to note that all the coefficients are significantly different from zero at the 5% level; moreover, the coefficients corresponding to public consumption are highly significant, confirming the existence of important long-run relationships between government consumption and private expenditures and indicating non-separability of consumer preferences between private and public goods.

The presence of substitutability or complementarity relationships between private and public consumption can be statistically verified by means of a LR test based on the log-likelihood values of the conditional and separable specifications. Whenever the null hypothesis of separability is not rejected, the substitutability/complementarity effects disappear and consumer demand depends only on relative prices and on total private

\[
\hat{\omega}_3 = 0.5474 - 0.1175 \log p_1 + 0.3072 \log p_2 + 0.1897 \log p_3 - 0.5098 \log (E_i / P_t) - 0.1565 G_t
\]

and

\[
\hat{\omega}_4 = 0.5930 - 0.0949 \log p_1 + 0.2507 \log p_2 + 0.1557 \log p_3 - 0.4407 \log (E_i / P_t) - 0.0944 G_t - 0.0797 G_t
\]

for the demand systems conditioned to G and GI+GC, respectively.

---

9 The third cointegrating share equations subject to homogeneity and symmetry constraints are equal to:
expenditure, and the effect of public consumption is limited to an income effect only. The results of LR tests\textsuperscript{10} for the two conditional specifications are reported in Table 7.

### Table 7 about here

The chi-squared statistics obtained show that the public provision of goods and services significantly affects the allocation of private expenditures. The hypothesis of separability is clearly rejected for all the specifications considered, with and without the theoretical restrictions imposed; in particular, the LR test values are higher than the 5 and 1 percent critical values and unambiguously indicate that consumer preferences are non-separable between publicly provided and privately purchased goods and services.

#### 4.3. Evaluating the relationships between private and public expenditure

The results presented in the previous section show the presence of significant long run relations in the conditional private demand system. The respect of the reduced rank condition and the high significance of the estimated long-run coefficients provide the ground for pursuing a more in-depth analysis and obtaining a quantitative measure of the effects exerted by government consumption on private spending. Before doing this, we reckon the representation of private demand behaviour in terms of income and price elasticities, distinguishing between conditional and separable specifications, to be useful for empirical comparison. The estimated elasticities, computed by using the homogeneity and symmetry restricted estimates, are presented in Table 8\textsuperscript{11}. The significant effects of the I(1) government variables found in the separability tests lead to

\textsuperscript{10} The tests have also been carried out by applying the small-sample correction [11]. However, since the adjusted test outcomes are analogous to the asymptotic results, they are not presented here.

\textsuperscript{11} Formally, income elasticity and Marshallian and Hicksian price elasticities for the conditional Almost Ideal model are as follows:

\[
\eta_i = 1 + \varphi_i / w_i \quad \text{(Income Elasticity)}
\]

\[
e_{ij}^M = \gamma_i / w_i + (1 - \eta_i)w_j - \delta_{ij} = \gamma_i / w_i - \varphi_i w_j / w_i - \delta_{ij} \quad \text{(Marshallian Price Elasticity)}
\]

\[
e_{ij}^H = \gamma_i / w_i + w_j - \delta_{ij} \quad \text{(Hicksian Price Elasticity)}
\]

where \( \delta_{ij} \) is Kronecker’s delta (equal to one if \( i = j \) and zero otherwise).
a cautious interpretation of the elasticity results of the separable specification, reported in Section (a) of Table 8. It is remarkable to point out that misspecification problems of separable specification can explain the unlikely negative value of the income elasticity of “Food, Energy and other non-durables”\textsuperscript{12}, while the expected dimension of elasticities is recovered only in the conditional model (Section (b) of Table 8). As it can be noted, the results seem to be, on the whole, quite plausible: income elasticities at the sample mean show that “Health, education, recreation and culture and social protection” (HER) are necessary goods. More important is the characteristic of necessary goods for “Food, Energy and Other non-durables”, with estimated values that are well below unity. On the contrary services are, as expected, luxury goods. High income elasticities (> 2) are suggested by the evidence that when the per-capita income is high, households value necessary goods relatively less than other private goods, such as services, and therefore they devote a low income share to necessary goods.

Table 8 about here

Of particular interest is the finding that price-elasticity estimates are all negative in the diagonal matrix and are significant at the five per cent level. As far as price elasticities are concerned, the compensated elasticity values are always lower than the uncompensated ones, as implied by the theoretical structure imposed on the parametric specification. In general, price elasticities are well defined and plausible.

Considering the HER and “Services” own-prices elasticities show an elastic response with an absolute value greater than 2, while the presence of food and energy into “Other non durables” determines an inelastic own-price response. Finally, the statistical significance of cross-price elasticity in private good demand leads to classify “Services” as substitutes of HER and “Other non-durables”, while the last two categories are complementary goods.

Moreover, we would like to stress the novelty of the results obtained by both verifying the assumption of non-separability and taking into account the simultaneous

\textsuperscript{12} These misspecification problems can also affect the results the cointegration rank test.
decomposition of private and public expenditures. This point can be appreciated by analysing Table 9 where the estimated elasticities of individual government expenditure (GI) and collective government expenditure (GC) are reported with respect to each private expenditure category of the demand system. Firstly, the results show that private consumption decisions are significantly affected by the public provision of goods and services and the public sector has simultaneous crowding out/in effects on the private sector. Secondly, individual and collective government expenditures (GI and GC) are more linked with some private goods and services and less with others. As expected, we note that the private counterpart of individual government expenditure (HRE) is characterized by the highest substitutability value. This estimation works against the claims that the private and public expenditure categories that produce similar utility cause a crowding-in effect (Kuehlwein, 1998; Fiorito and Kollintzas, 2004). The classical crowding out hypothesis is confirmed: public expenditure has a large negative contemporaneous influence on private consumer spending on health, education, recreation and social protection. Moreover, the elasticities of “Other non durables” with respect to government spending are significantly different from zero and negative (the ranges are -0.128/-0.303 and -0.124/-0.223, respectively for GI and GC), thus indicating, as above, a substitution relationship between private expenditure and public consumption.

The elasticity of the “Services” category, with respect to GI and GC, show that an increase in the public provision of goods and services causes an increase in private spending due to the presence of a positive income effect; this strengthens the complementarity effect exerted by public consumption expenditures and causes a shift of consumer preferences. The results are in line with the findings of Karras (1994), Kuehlwein (1998) and Fiorito and Kollintzas (2004) concerning the possibility of having complementarity relationships between the public and private sectors.

*Table 9 about here*

From the analysis of the dynamics of the estimated elasticities taken at each decade, it is possible to note that the elasticity of substitution of the HRE expenditures with
respect to GI increases in absolute value (from -0.631 in 1960 to -0.708 in 2000) while the impact of GC decreases (from -0.724 to -0.602). Concerning non-durable goods, GI and GC display an upward trend over the entire sample period, doubling the initial substitutability values. On the contrary, the elasticity of services with respect to government expenditure shows a slight downward trend for GC, while the impact of GI remains quite stable over the sample.

In order to compare our findings to those obtained by the traditional crowding-out literature, we compute the average effects of GI and GC as a mean of the elasticities for the three private consumption categories at each decade. A summary of the results is provided in the last two columns of Table 9. Although the multivariate approach proposed is different from that in standard macroeconomic literature, it is worth noting that the range of estimated elasticities ($GI_{1970} = -0.130$, $GI_{2000} = -0.204$; $GC_{1970} = -0.156$, $GC_{2000} = -0.170$) is very similar to that found in previous analyses under different estimation procedures, sampling and specification (see Darby and Malley, 1996, p. 139), confirming the validity of our findings.

5. Conclusion

In this paper we have used a micro-based approach to investigate the relationships between disaggregate private consumption and publicly provided goods. The different impact of individual and collective government expenditures on private utility function has led to extend the analysis to account for these heterogeneous behaviours. Thus, a conditional demand system is derived and government expenditure or its components have been used as a rationed good. The novelty of our analysis, with respect to the traditional literature, concerns the possibility of simultaneously testing for the presence of crowding out/in effects in a conditional demand system by verifying the non-separability hypothesis of individual preferences. To do this we have assumed that government expenditure, or its components, was exogenous I(1) forcing variables and

---

13 Since each estimated parameter $\theta_{ij}$ is weighted by the budget share, the average substitution elasticities are directly obtained.
the long run conditional demand system specified in a multivariate ECM to be identified by the cointegration rank.

Three main empirical implications have been derived estimating a conditional long-run demand system for the UK in the 1964:01 – 2002:04 period.

Private consumption decisions are significantly affected by the public provision of goods. Moreover, the decomposition of the private expenditure in three different budget shares leads to simultaneous crowding out/in effects. Secondly, we cannot reject the traditional point of view according to which government expenditure crowds out private goods and services. These findings are in marked contrast with recent empirical evidence of complementary effects. On the other hand, income effects generate a positive impact of government expenditure on the “Services” category. Thirdly, we have investigated the robustness of our estimation results with respect to previous works by means of an aggregate indicator. The average value of public consumption elasticity is found to be close to the range of previous literature showing the presence of a significant substitutability relationship.

On the basis of our results, the suggestion is that efficient fiscal policy planning should take into account the heterogeneous impacts of government spending on the allocation of private expenditures. The future needs that the public sector is likely to have to face are strictly connected with the behavioural reactions of economic agents to changes in public expenditure and taxation.
References


### Tables

#### Table 1 – Unit root tests

<table>
<thead>
<tr>
<th>Test specification</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( w_3 )</th>
<th>( \log p_1 )</th>
<th>( \log p_2 )</th>
<th>( \log p_3 )</th>
<th>Income</th>
<th>G</th>
<th>GI</th>
<th>GC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{intercept+trend} )</td>
<td>-2.609</td>
<td>-1.095</td>
<td>-0.971</td>
<td>0.165</td>
<td>-0.593</td>
<td>-0.247</td>
<td>-2.831</td>
<td>1.563</td>
<td>-2.046</td>
<td>0.189</td>
</tr>
<tr>
<td>( \text{intercept} )</td>
<td>0.186</td>
<td>-1.002</td>
<td>0.509</td>
<td>-2.354</td>
<td>-1.557</td>
<td>-2.057</td>
<td>0.425</td>
<td>0.533</td>
<td>0.512</td>
<td>[0.971, 0.752, 0.987, 0.157, 0.502, 0.386, 0.984, 0.987, 0.987, 0.892]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test specification</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( w_3 )</th>
<th>( \log p_1 )</th>
<th>( \log p_2 )</th>
<th>( \log p_3 )</th>
<th>Income</th>
<th>G</th>
<th>GI</th>
<th>GC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{intercept+trend} )</td>
<td>-2.407</td>
<td>-1.027</td>
<td>-1.393</td>
<td>0.928</td>
<td>0.256</td>
<td>0.489</td>
<td>-2.414</td>
<td>1.377</td>
<td>-2.474</td>
<td>-1.507</td>
</tr>
<tr>
<td>( \text{intercept} )</td>
<td>0.521</td>
<td>-0.875</td>
<td>-0.538</td>
<td>-2.597</td>
<td>-1.983</td>
<td>-2.249</td>
<td>0.242</td>
<td>0.538</td>
<td>0.410</td>
<td>[0.987, 0.794, 0.879, 0.096, 0.294, 0.190, 0.975, 0.988, 0.983, 0.928]</td>
</tr>
</tbody>
</table>

**Notes:** figures in normal parentheses denote the number of lagged dependent variables in the ADF test equation and the Newey-West bandwidth for the PP test, respectively. Figures in squared brackets are MacKinnon (1996) one-sided p-values.
Table 2 – Lag order selection

**a) Demand system conditional on G**

<table>
<thead>
<tr>
<th>Order</th>
<th>LL</th>
<th>AIC</th>
<th>SBC</th>
<th>Adjusted LR test</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3721.5</td>
<td>3493.5</td>
<td>3151.9</td>
<td>-----</td>
</tr>
<tr>
<td>5</td>
<td>3701.1</td>
<td>3509.1</td>
<td>3221.3</td>
<td>$\chi^2(36)=30.4567 \ [0.729]$</td>
</tr>
<tr>
<td>4</td>
<td>3667.8</td>
<td>3511.8</td>
<td>3278.1</td>
<td>$\chi^2(72)=79.8194 \ [0.247]$</td>
</tr>
<tr>
<td>3</td>
<td>3645.1</td>
<td>3525.1</td>
<td>3345.2</td>
<td>$\chi^2(108)=113.6829 \ [0.335]$</td>
</tr>
<tr>
<td>2</td>
<td>3619.1</td>
<td>3535.1</td>
<td>3409.2</td>
<td>$\chi^2(144)=152.3419 \ [0.301]$</td>
</tr>
<tr>
<td>1</td>
<td>3557.5</td>
<td>3509.5</td>
<td>3437.5</td>
<td>$\chi^2(180)=243.8907 \ [0.001]$</td>
</tr>
<tr>
<td>0</td>
<td>1772</td>
<td>1760</td>
<td>1742</td>
<td>$\chi^2(216)=2898.0 \ [0.000]$</td>
</tr>
</tbody>
</table>

**b) Demand system conditional on GI and GC**

<table>
<thead>
<tr>
<th>Order</th>
<th>LL</th>
<th>AIC</th>
<th>SBC</th>
<th>Adjusted LR test</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3726.8</td>
<td>3486.8</td>
<td>3127.1</td>
<td>-----</td>
</tr>
<tr>
<td>5</td>
<td>3705.1</td>
<td>3501.1</td>
<td>3195.4</td>
<td>$\chi^2(36)=31.6760 \ [0.674]$</td>
</tr>
<tr>
<td>4</td>
<td>3671.2</td>
<td>3503.2</td>
<td>3251.4</td>
<td>$\chi^2(72)=81.1657 \ [0.215]$</td>
</tr>
<tr>
<td>3</td>
<td>3648</td>
<td>3516</td>
<td>3318.2</td>
<td>$\chi^2(108)=114.9255 \ [0.306]$</td>
</tr>
<tr>
<td>2</td>
<td>3622.3</td>
<td>3526.3</td>
<td>3382.4</td>
<td>$\chi^2(144)=152.5313 \ [0.297]$</td>
</tr>
<tr>
<td>1</td>
<td>3561.9</td>
<td>3501.9</td>
<td>3412</td>
<td>$\chi^2(180)=240.5958 \ [0.002]$</td>
</tr>
<tr>
<td>0</td>
<td>1936.5</td>
<td>1912.5</td>
<td>1876.5</td>
<td>$\chi^2(216)=2612.9 \ [0.000]$</td>
</tr>
</tbody>
</table>

**Notes:** AIC=Akaike Information Criterion, SBC=Schwarz Bayesian Criterion. Figures in round parentheses denote the degrees of freedom of the chi-squared statistics. Figures in squared brackets are adjusted LR test $p$-values.
### Table 3 – Johansen’s Cointegration Rank Tests

**a) Demand system conditional on G**

<table>
<thead>
<tr>
<th>No. of CE(s)</th>
<th>Max-Eigen Statistic</th>
<th>95% critical value</th>
<th>90% critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>$r = 0$</td>
<td>80.389</td>
<td>43.76</td>
</tr>
<tr>
<td>$H_1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td></td>
<td>47.966</td>
<td>37.48</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td></td>
<td>30.542</td>
<td>31.48</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td></td>
<td>15.272</td>
<td>25.54</td>
</tr>
<tr>
<td>$r \leq 4$</td>
<td></td>
<td>6.845</td>
<td>18.88</td>
</tr>
<tr>
<td>$r \leq 5$</td>
<td></td>
<td>5.787</td>
<td>12.45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No. of CE(s)</th>
<th>Trace Statistic</th>
<th>95% critical value</th>
<th>90% critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>$r = 0$</td>
<td>186.801</td>
<td>116.30</td>
</tr>
<tr>
<td>$H_1$</td>
<td>$r \geq 1$</td>
<td>106.412</td>
<td>86.58</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$r \geq 2$</td>
<td>58.446</td>
<td>62.75</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>$r \geq 3$</td>
<td>27.904</td>
<td>42.40</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>$r \geq 4$</td>
<td>12.632</td>
<td>25.23</td>
</tr>
<tr>
<td>$r \leq 4$</td>
<td>$r \geq 5$</td>
<td>5.787</td>
<td>12.45</td>
</tr>
</tbody>
</table>

**b) Demand system conditional on GI and GC**

<table>
<thead>
<tr>
<th>No. of CE(s)</th>
<th>Max-Eigen Statistic</th>
<th>95% critical value</th>
<th>90% critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>$r = 0$</td>
<td>80.700</td>
<td>46.90</td>
</tr>
<tr>
<td>$H_1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td></td>
<td>51.480</td>
<td>40.57</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td></td>
<td>29.538</td>
<td>34.69</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td></td>
<td>17.053</td>
<td>28.49</td>
</tr>
<tr>
<td>$r \leq 4$</td>
<td></td>
<td>7.130</td>
<td>21.92</td>
</tr>
<tr>
<td>$r \leq 5$</td>
<td></td>
<td>4.944</td>
<td>15.27</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No. of CE(s)</th>
<th>Trace Statistic</th>
<th>95% critical value</th>
<th>90% critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>$r = 0$</td>
<td>190.845</td>
<td>128.93</td>
</tr>
<tr>
<td>$H_1$</td>
<td>$r \geq 1$</td>
<td>110.145</td>
<td>97.57</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$r \geq 2$</td>
<td>58.664</td>
<td>72.15</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>$r \geq 3$</td>
<td>29.127</td>
<td>49.43</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>$r \geq 4$</td>
<td>12.074</td>
<td>30.46</td>
</tr>
<tr>
<td>$r \leq 4$</td>
<td>$r \geq 5$</td>
<td>4.944</td>
<td>15.27</td>
</tr>
</tbody>
</table>

**Notes:** the 95 and 90 percent critical values used for the rank cointegration test with exogenous I(1) are taken from Harbo et al. (1998).

### Table 4 – Estimated cointegrating vectors subject to exact identifying restrictions

**a) demand system conditional on G**

<table>
<thead>
<tr>
<th></th>
<th>$w_{1t}$</th>
<th>$w_{2t}$</th>
<th>log $p_1$</th>
<th>log $p_2$</th>
<th>log $p_3$</th>
<th>log $(E_t/P_t)$</th>
<th>GI</th>
<th>intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cointegrating Vector 1</td>
<td>-1</td>
<td>0</td>
<td>-0.0755</td>
<td>0.2511</td>
<td>-0.1681</td>
<td>-0.0935</td>
<td>0.2077</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.059)</td>
<td>(0.062)</td>
<td>(0.047)</td>
<td>(0.046)</td>
<td>(0.049)</td>
<td></td>
</tr>
<tr>
<td>Cointegrating Vector 2</td>
<td>0</td>
<td>-1</td>
<td>0.2764</td>
<td>-0.4013</td>
<td>0.1501</td>
<td>0.3987</td>
<td>0.1494</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.132)</td>
<td>(0.144)</td>
<td>(0.106)</td>
<td>(0.105)</td>
<td>(0.114)</td>
<td></td>
</tr>
</tbody>
</table>

**b) demand system conditional on GI and GC**

<table>
<thead>
<tr>
<th></th>
<th>$w_{1t}$</th>
<th>$w_{2t}$</th>
<th>log $p_1$</th>
<th>log $p_2$</th>
<th>log $p_3$</th>
<th>log $(E_t/P_t)$</th>
<th>GI</th>
<th>GC</th>
<th>intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cointegrating Vector 1</td>
<td>-1</td>
<td>0</td>
<td>-0.0537</td>
<td>0.2121</td>
<td>-0.1498</td>
<td>-0.0515</td>
<td>-0.0675</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.042)</td>
<td>(0.040)</td>
<td>(0.034)</td>
<td>(0.031)</td>
<td>(0.025)</td>
<td>(0.020)</td>
<td></td>
</tr>
<tr>
<td>Cointegrating Vector 2</td>
<td>0</td>
<td>-1</td>
<td>0.2686</td>
<td>-0.3623</td>
<td>0.1175</td>
<td>0.3293</td>
<td>0.1047</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.108)</td>
<td>(0.105)</td>
<td>(0.090)</td>
<td>(0.081)</td>
<td>(0.067)</td>
<td>(0.052)</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** standard errors in round brackets.
### Table 5 – Tests for theoretical restrictions

<table>
<thead>
<tr>
<th>Model Specification</th>
<th>Standard asymptotic results</th>
<th>Small-sample correction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LR statistic [ \chi^2 ]</td>
<td>p-value</td>
</tr>
<tr>
<td></td>
<td>Test statistic</td>
<td>5% critical value</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1% critical value</td>
</tr>
<tr>
<td>1. Conditional specifications:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) G</td>
<td>13.140</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.944</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.509</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12.488</td>
</tr>
<tr>
<td>b) GI+GC</td>
<td>14.742</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11.507</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.617</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12.649</td>
</tr>
<tr>
<td>2. Separable specification</td>
<td>20.780</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17.623</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.403</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12.331</td>
</tr>
</tbody>
</table>

**Notes:** small-sample corrected critical values are computed as: 
\[ K = nT \log[1 + \rho^2_{k-1} / (nT - k)] \]

### Table 6 – Estimated cointegrating vectors with homogeneity and symmetry restrictions imposed

#### a) demand system conditional on G

<table>
<thead>
<tr>
<th></th>
<th>( w_{2t} )</th>
<th>( w_{2t} )</th>
<th>( \log p_1 )</th>
<th>( \log p_2 )</th>
<th>( \log p_3 )</th>
<th>( \log (E_t / P_t) )</th>
<th>G</th>
<th>intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cointegrating Vector 1</strong></td>
<td>-1</td>
<td>0</td>
<td>-0.1593</td>
<td>0.2768</td>
<td>-0.1175</td>
<td>-0.0539</td>
<td>-0.1432</td>
<td>0.2435</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.053)</td>
<td>(0.071)</td>
<td>(0.036)</td>
<td>(0.036)</td>
<td>(0.051)</td>
<td>(0.052)</td>
</tr>
<tr>
<td><strong>Cointegrating Vector 2</strong></td>
<td>0</td>
<td>-1</td>
<td>0.2768</td>
<td>-0.5840</td>
<td>0.3072</td>
<td>0.5637</td>
<td>0.2997</td>
<td>0.2091</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.071)</td>
<td>(0.170)</td>
<td>(0.124)</td>
<td>(0.123)</td>
<td>(0.096)</td>
<td>(0.097)</td>
</tr>
</tbody>
</table>

**Notes:** standard errors in round brackets.

#### b) demand system conditional on GI and GC

<table>
<thead>
<tr>
<th></th>
<th>( w_{2t} )</th>
<th>( w_{2t} )</th>
<th>( \log p_1 )</th>
<th>( \log p_2 )</th>
<th>( \log p_3 )</th>
<th>( \log (E_t / P_t) )</th>
<th>GI</th>
<th>GC</th>
<th>intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cointegrating Vector 1</strong></td>
<td>-1</td>
<td>0</td>
<td>-0.1155</td>
<td>0.2104</td>
<td>-0.0949</td>
<td>-0.0072</td>
<td>-0.0640</td>
<td>-0.0628</td>
<td>0.2144</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.031)</td>
<td>(0.041)</td>
<td>(0.025)</td>
<td>(0.024)</td>
<td>(0.023)</td>
<td>(0.018)</td>
<td>(0.037)</td>
</tr>
<tr>
<td><strong>Cointegrating Vector 2</strong></td>
<td>0</td>
<td>-1</td>
<td>0.2104</td>
<td>-0.4610</td>
<td>0.2507</td>
<td>0.4479</td>
<td>0.1584</td>
<td>0.1425</td>
<td>0.1926</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.041)</td>
<td>(0.124)</td>
<td>(0.104)</td>
<td>(0.100)</td>
<td>(0.079)</td>
<td>(0.043)</td>
<td>(0.103)</td>
</tr>
</tbody>
</table>

**Notes:** standard errors in round brackets.
### Table 7 – Separability tests

**H$_0$: Separability between private and public consumption**

<table>
<thead>
<tr>
<th>Model Specification</th>
<th>LR statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Unrestricted specification</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) G</td>
<td>62.40 (14)</td>
<td>0.000</td>
</tr>
<tr>
<td>b) GI+GC</td>
<td>51.60 (28)</td>
<td>0.004</td>
</tr>
<tr>
<td><strong>2. Restricted specification</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) G</td>
<td>54.60 (14)</td>
<td>0.000</td>
</tr>
<tr>
<td>b) GI+GC</td>
<td>45.40 (28)</td>
<td>0.020</td>
</tr>
</tbody>
</table>

*Notes:* the degrees of freedom of each $\chi^2$ statistic are reported in round brackets.

### Table 8 – Estimated price and income elasticities

<table>
<thead>
<tr>
<th>Private expenditures</th>
<th>Price Elasticities</th>
<th>Income Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Marshallian</td>
<td>Hicksian</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>a) Separable specification</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) HER</td>
<td>-1.202 (0.296)</td>
<td>-1.135 (0.348)</td>
</tr>
<tr>
<td>(2) Services</td>
<td>0.110 (0.136)</td>
<td>0.340 (0.132)</td>
</tr>
<tr>
<td>(3) Other non-durables</td>
<td>-0.052 (0.092)</td>
<td>-0.074 (0.088)</td>
</tr>
<tr>
<td>b) Conditional to GI and GC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) HER</td>
<td>-2.296 (0.349)</td>
<td>-2.215 (0.464)</td>
</tr>
<tr>
<td>(2) Services</td>
<td>0.421 (0.106)</td>
<td>0.608 (0.307)</td>
</tr>
<tr>
<td>(3) Other non-durables</td>
<td>-0.110 (0.053)</td>
<td>-0.099 (0.206)</td>
</tr>
</tbody>
</table>

*Notes:* asymptotic standard errors in round brackets
<table>
<thead>
<tr>
<th>Year</th>
<th>HER</th>
<th>Services</th>
<th>Other non-durables</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GI</td>
<td>GC</td>
<td>GI</td>
<td>GC</td>
</tr>
<tr>
<td>1970</td>
<td>-0.631</td>
<td>-0.724</td>
<td>0.368</td>
<td>0.381</td>
</tr>
<tr>
<td></td>
<td>(0.225)</td>
<td>(0.202)</td>
<td>(0.193)</td>
<td>(0.124)</td>
</tr>
<tr>
<td>1980</td>
<td>-0.674</td>
<td>-0.746</td>
<td>0.402</td>
<td>0.405</td>
</tr>
<tr>
<td></td>
<td>(0.244)</td>
<td>(0.213)</td>
<td>(0.207)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>1990</td>
<td>-0.722</td>
<td>-0.695</td>
<td>0.375</td>
<td>0.330</td>
</tr>
<tr>
<td></td>
<td>(0.261)</td>
<td>(0.200)</td>
<td>(0.189)</td>
<td>(0.100)</td>
</tr>
<tr>
<td>2000</td>
<td>-0.708</td>
<td>-0.602</td>
<td>0.401</td>
<td>0.314</td>
</tr>
<tr>
<td></td>
<td>(0.265)</td>
<td>(0.176)</td>
<td>(0.194)</td>
<td>(0.094)</td>
</tr>
</tbody>
</table>

*Notes: asymptotic standard errors in round brackets*